Probability and Random Processes

ECS 315

Asst. Prof. Dr. Prapun Suksompong

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10 Continuous Random Variables

Office Hours:

Check Google Calendar on the course website.

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6th floor of Sirindhralai building, BKD

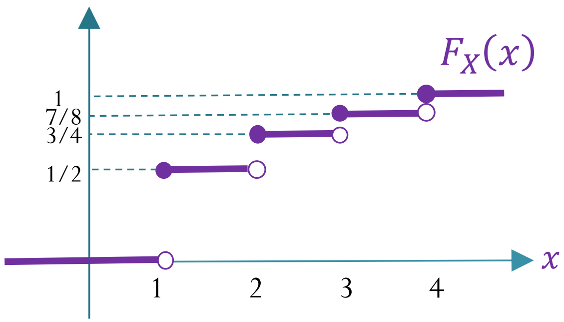
Sections 10.1-10.2

Discrete RV

- pmf: $p_X(x) \equiv P[X = x]$
  - Two characterizing properties:
    - $p_X(x) \geq 0$
    - $\sum_x p_X(x) = 1$
- $S_X = \{x: p_X(x) > 0\}$
- $P[\text{some statement(s) about } X]$

$$= \sum_{\substack{\{\text{all the } x \text{ values that} \\ \text{satisfy the statement(s)}\}}} p_X(x)$$

- cdf is a staircase function with jumps whose size at $x = c$ gives $P[X = c]$.

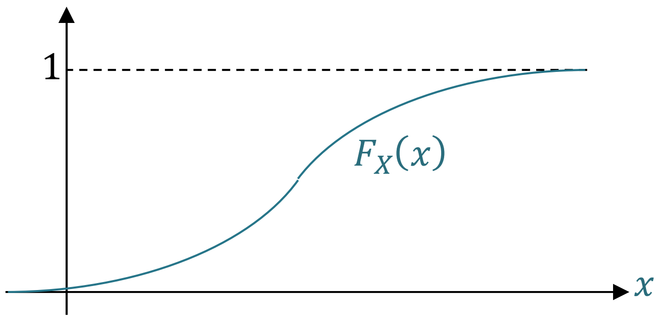


Continuous RV

- $P[X = x] = 0$
- pdf : $P[x_0 \leq x \leq x_0 + \Delta x] \approx f_X(x_0)\Delta x$ (probability per unit length: $f_X(x_0)$)
  - Two characterizing properties:
    - $f_X(x) \geq 0$
    - $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$
- $S_X = \{x: f_X(x) > 0\}$
- $P[\text{some statement(s) about } X] =$

$$\int_{\substack{\{\text{all the } x \text{ values that} \\ \text{satisfy the statement(s)}\}}} f_X(x)dx$$

- cdf is a continuous function.



Chapter 9 vs. Section 10.3

Discrete RV	Continuous RV
$\mathbb{E}X = \sum_{x} x p_X(x)$	$\mathbb{E}X = \int_{-\infty}^{\infty} x f_X(x) dx$
$\mathbb{E}[g(X)] = \sum_{x} g(x) p_X(x)$	$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$
$\mathbb{E}[X^2] = \sum_{x} x^2 p_X(x)$	$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x) dx$

$$\operatorname{Var}[X] = \mathbb{E}\left[(X - \mathbb{E}X)^2\right] = \mathbb{E}[X^2] - (\mathbb{E}X)^2$$

$$\sigma_X = \sqrt{\operatorname{Var}[X]}$$

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10.1 Probability Density Function

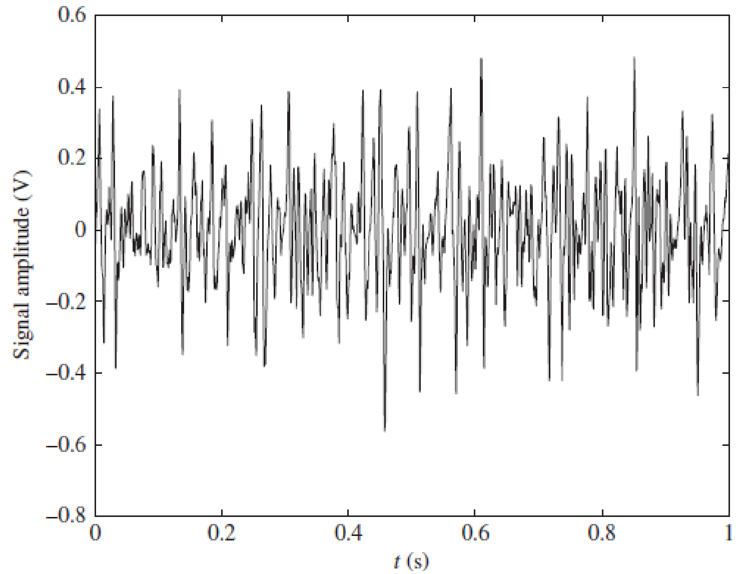
Ex. `rand` function

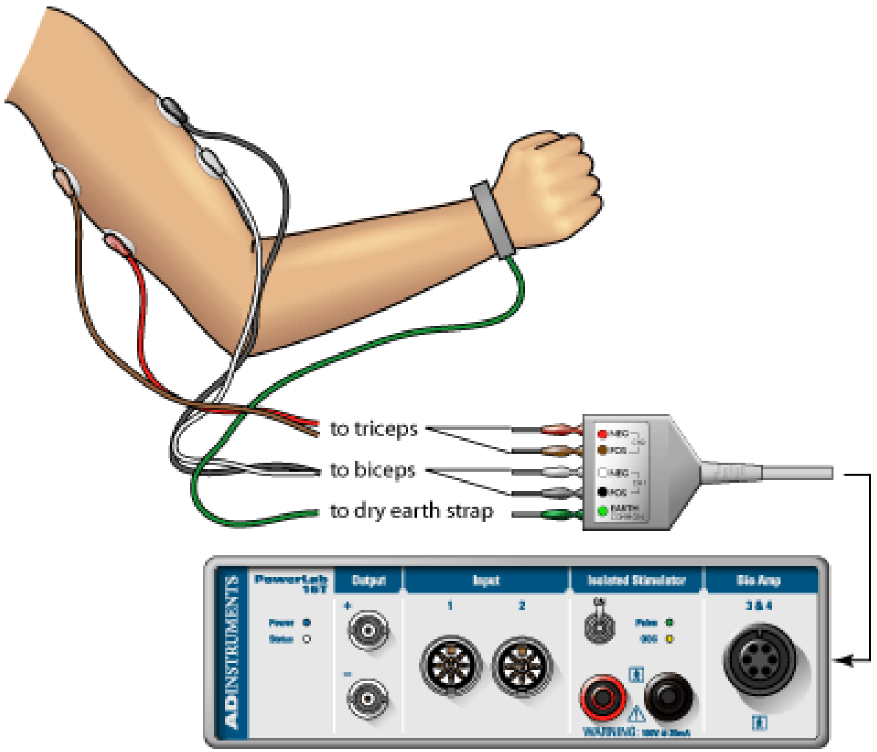
- Generate an array of uniformly distributed pseudorandom numbers.
  - The pseudorandom values are drawn from the **standard uniform distribution** on the open **interval (0,1)**.
- `rand` returns a scalar.
- `rand(m,n)` or `rand([m,n])` returns an *m*-by-*n* matrix.
  - `rand(n)` returns an *n*-by-*n* matrix

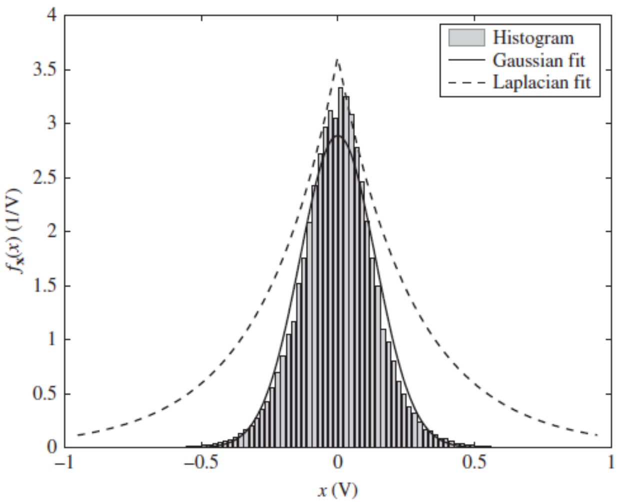
```
>> rand

ans =

    0.3816

>> rand(10,2)

ans =

    0.7655    0.6551
    0.7952    0.1626
    0.1869    0.1190
    0.4898    0.4984
    0.4456    0.9597
    0.6463    0.3404
    0.7094    0.5853
    0.7547    0.2238
    0.2760    0.7513
    0.6797    0.2551
```

Ex. Muscle Activity

- Look at electrical activity of skeletal muscle by recording a human electromyogram (EMG).





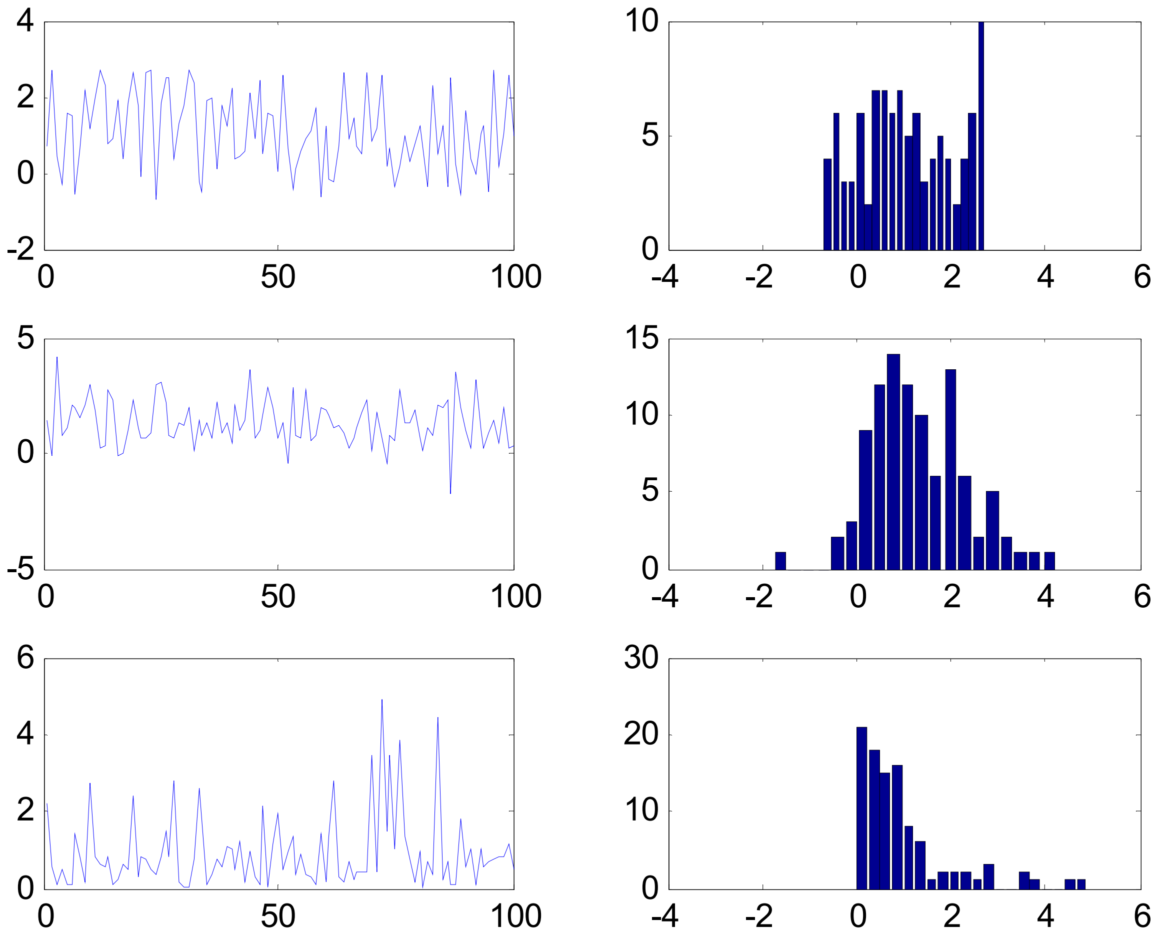


[http://www.adinstruments.com/solutions/education/ltexp/electromyography-emg-german]

Three Important Continuous RVs

```
close all; clear all;
N = 1e6; b = 20; m = 1; s = 1;
R = [1-5*s,1+5*s];
% Uniform
X = (2*sqrt(3)*(rand(1,N)-0.5))+1;
subplot(3,2,1); plot(X);
subplot(3,2,2); plotHistPdf(X,b)
xlim(R)
% Normal
X = randn(1,N)+1;
subplot(3,2,3); plot(X);
subplot(3,2,4); plotHistPdf(X,b)
xlim(R)
% Exponential
X = exprnd(1,1,N);
subplot(3,2,5); plot(X);
subplot(3,2,6); plotHistPdf(X,b)
xlim(R)
```

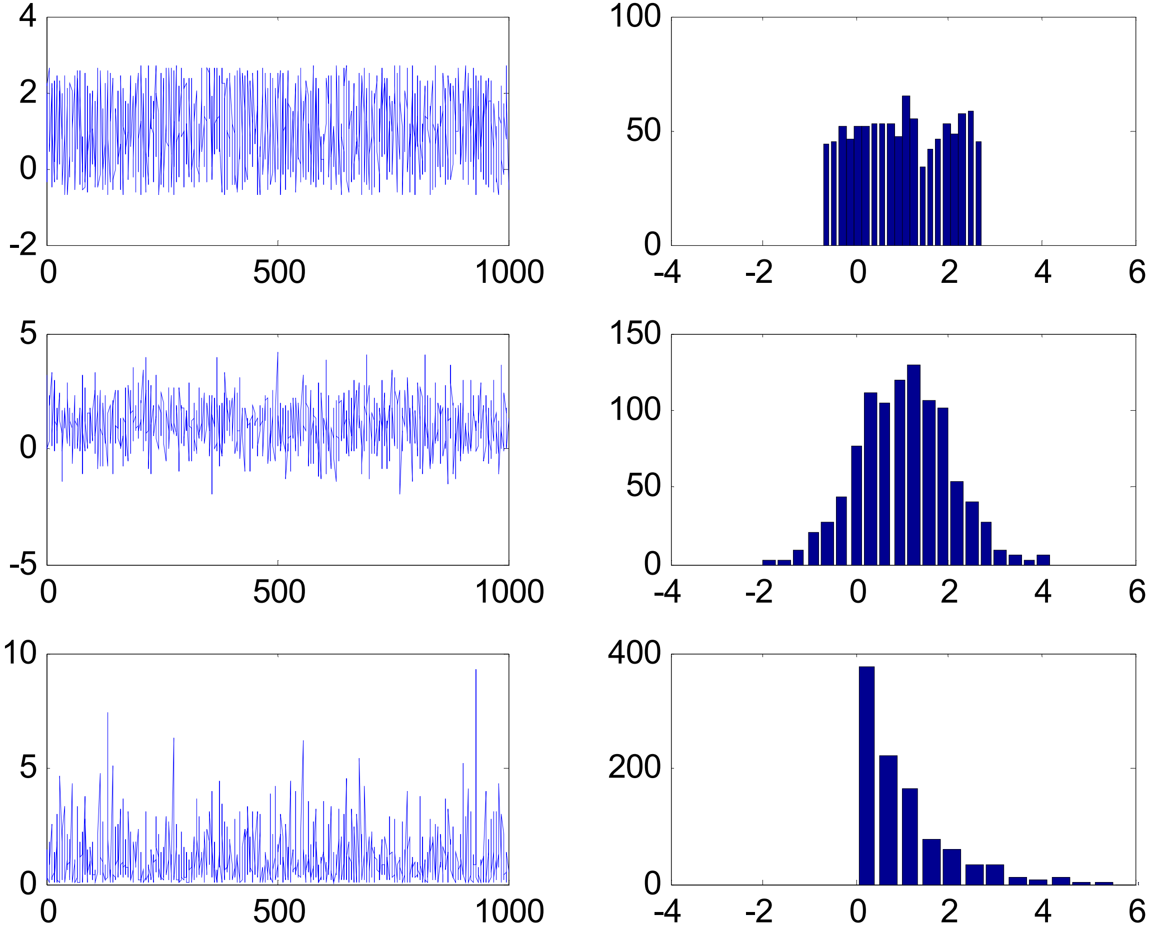
Three Important Continuous RVs



Mean = 1
Std = 1
N = 100

[IntroThreeContinuousRV.m]

Three Important Continuous RVs

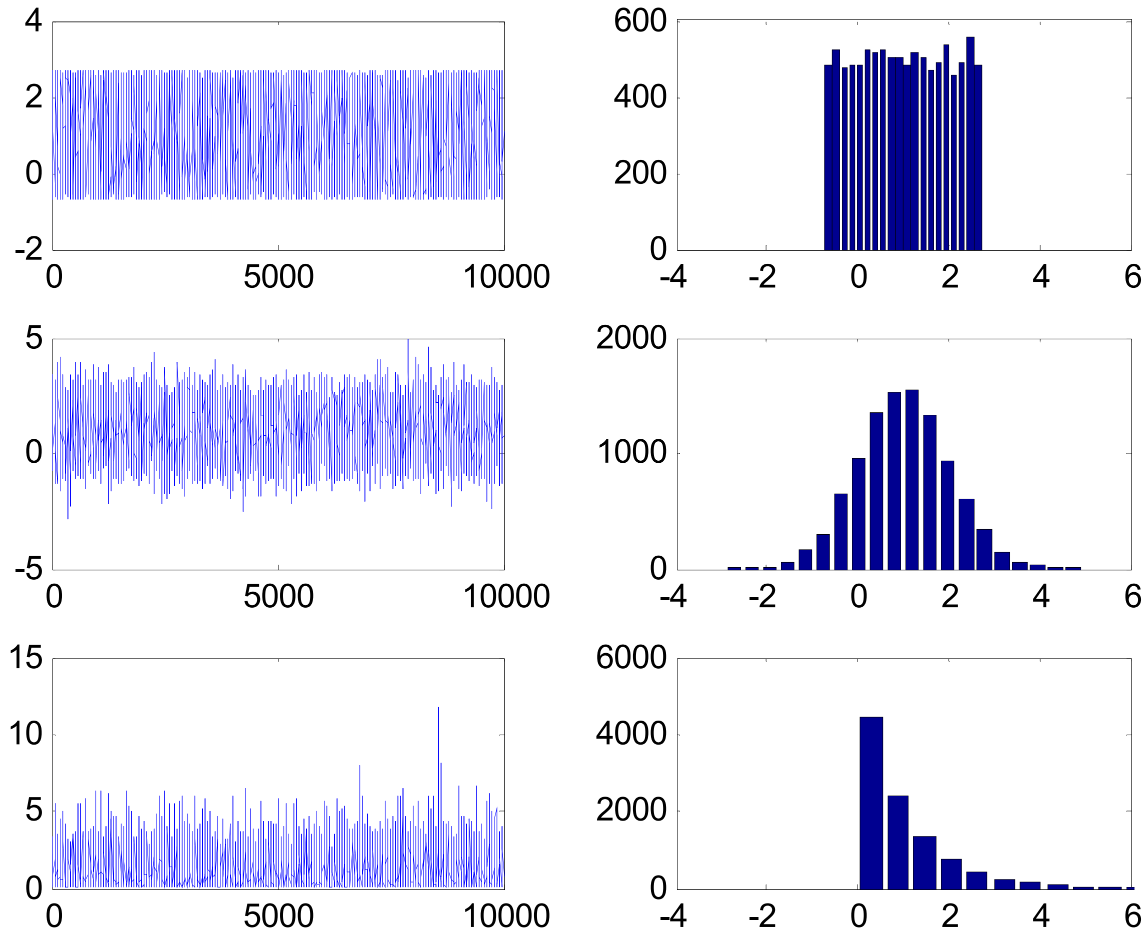


Mean = 1

Std = 1

N = 1,000

Three Important Continuous RVs



Mean = 1

Std = 1

N = 10,000

Review: P[some condition(s) on X]

For discrete random variable,

8.14. Steps to find probability of the form P [some condition(s) on X] when the pmf $p_X(x)$ is known.

(a) Find the support of X.

(b) Consider only the x inside the support. Find all values of x that satisfy the condition(s).

(c) Evaluate the pmf at x found in the previous step.

(d) Add the pmf values from the previous step.

$$P[\text{some condition(s) on } X] = \sum p_X(x)$$

Discrete RV

Sum over all the x values that satisfy the condition(s)

P[some condition(s) on X]

- **For discrete random variable,**

$$P[\text{some condition(s) on } X] = \sum \overbrace{p_X(x)}^{\text{probability mass function (pmf)}}$$

Discrete RV

Sum over all the x values that satisfy the condition(s)

- **For continuous random variable,**

pmf → pdf

$\sum \rightarrow \int$

$$P[\text{some condition(s) on } X] = \int \overbrace{f_X(x)}^{\text{probability density function (pdf)}} dx$$

Continuous RV

Integrate over all the x values that satisfy the condition(s)

Support of a RV

- In general, the **support** of a RV X is any set S such that

$$P[X \in S] = 1.$$

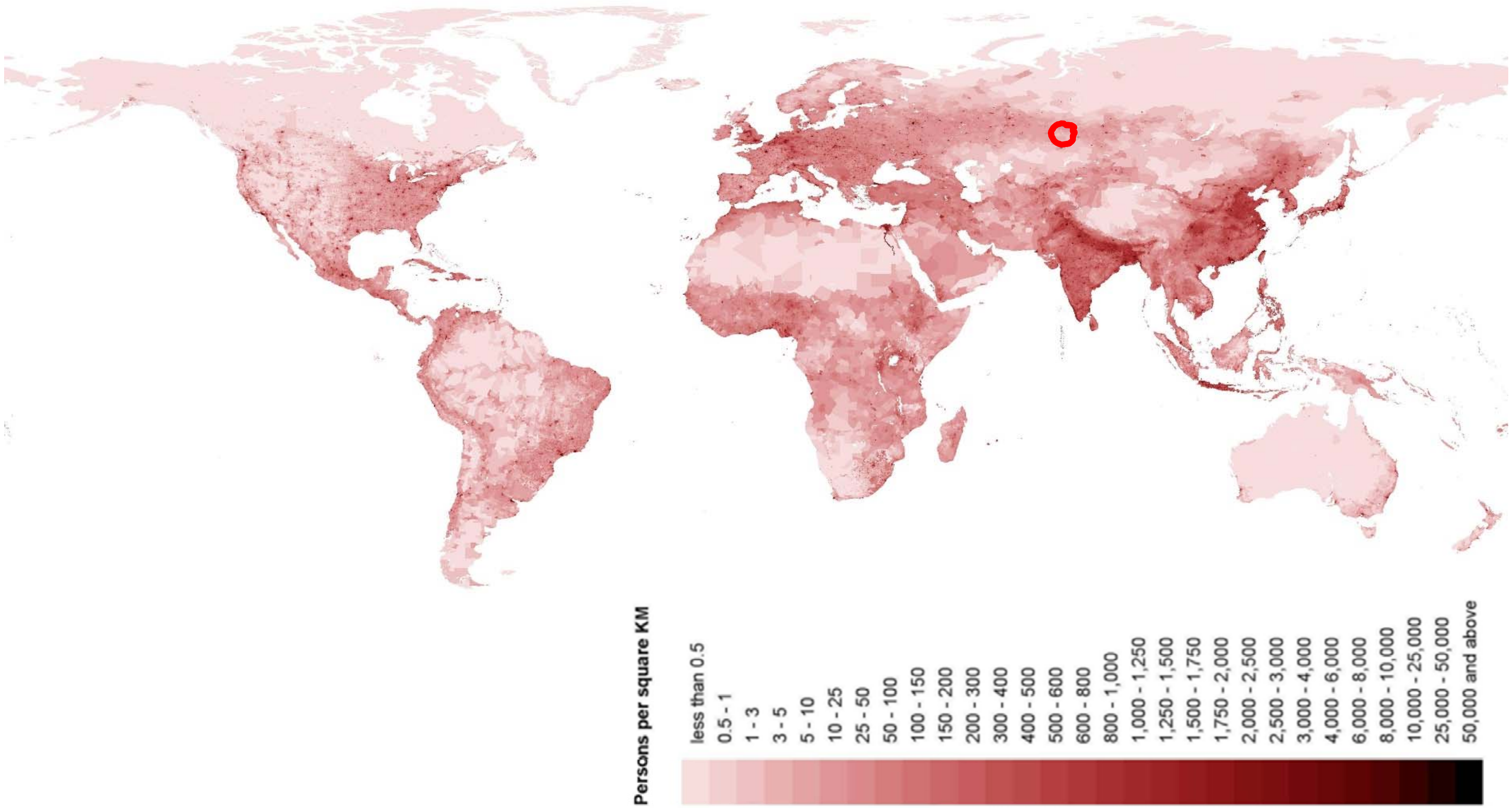
- In this class, we try to find the smallest (minimal) set that works as a support.
- **For discrete random variable,**

$$S_X = \{x: p_X(x) > 0\}$$

- **For continuous random variable,**

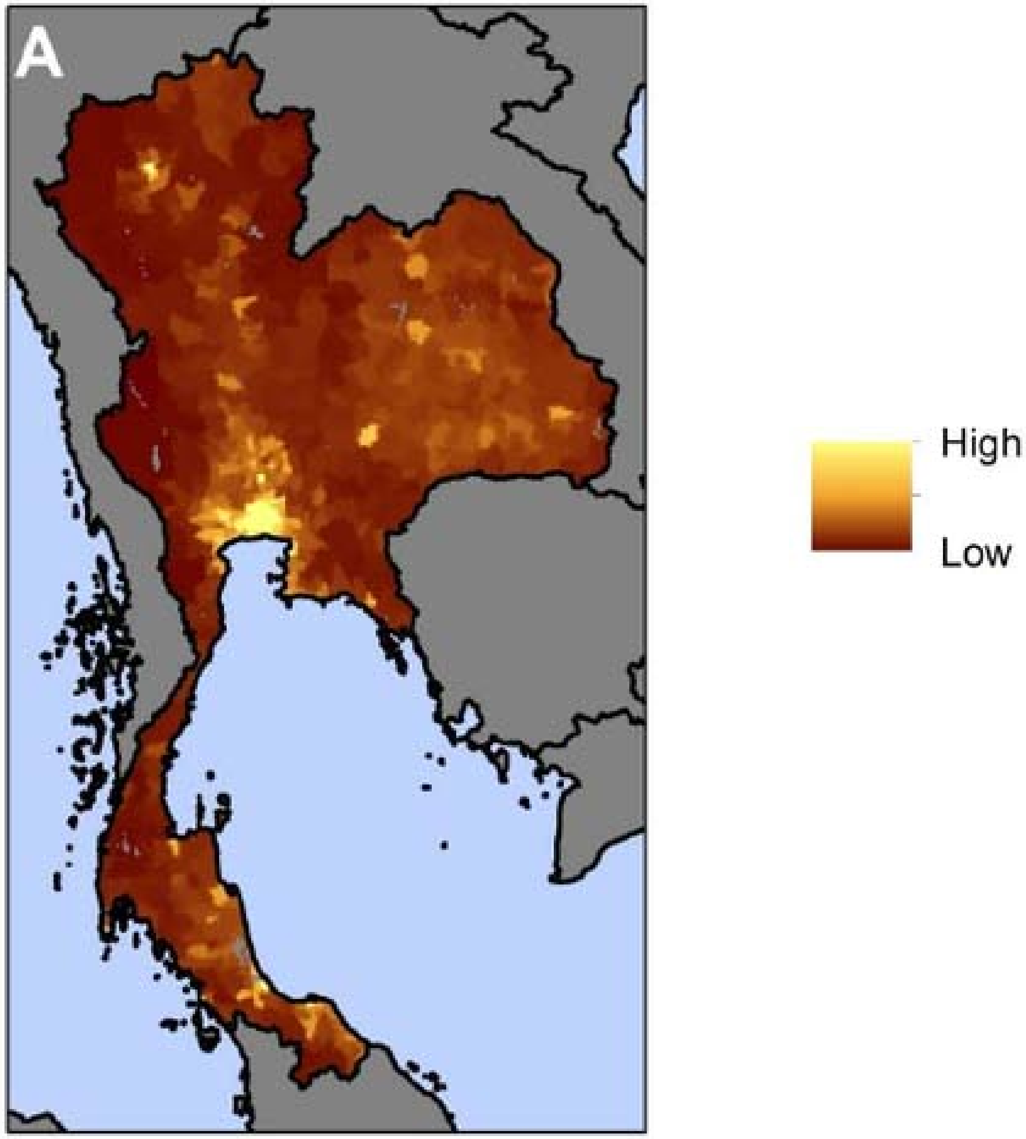
$$S_X = \{x: f_X(x) > 0\}$$

World Map of Population Density



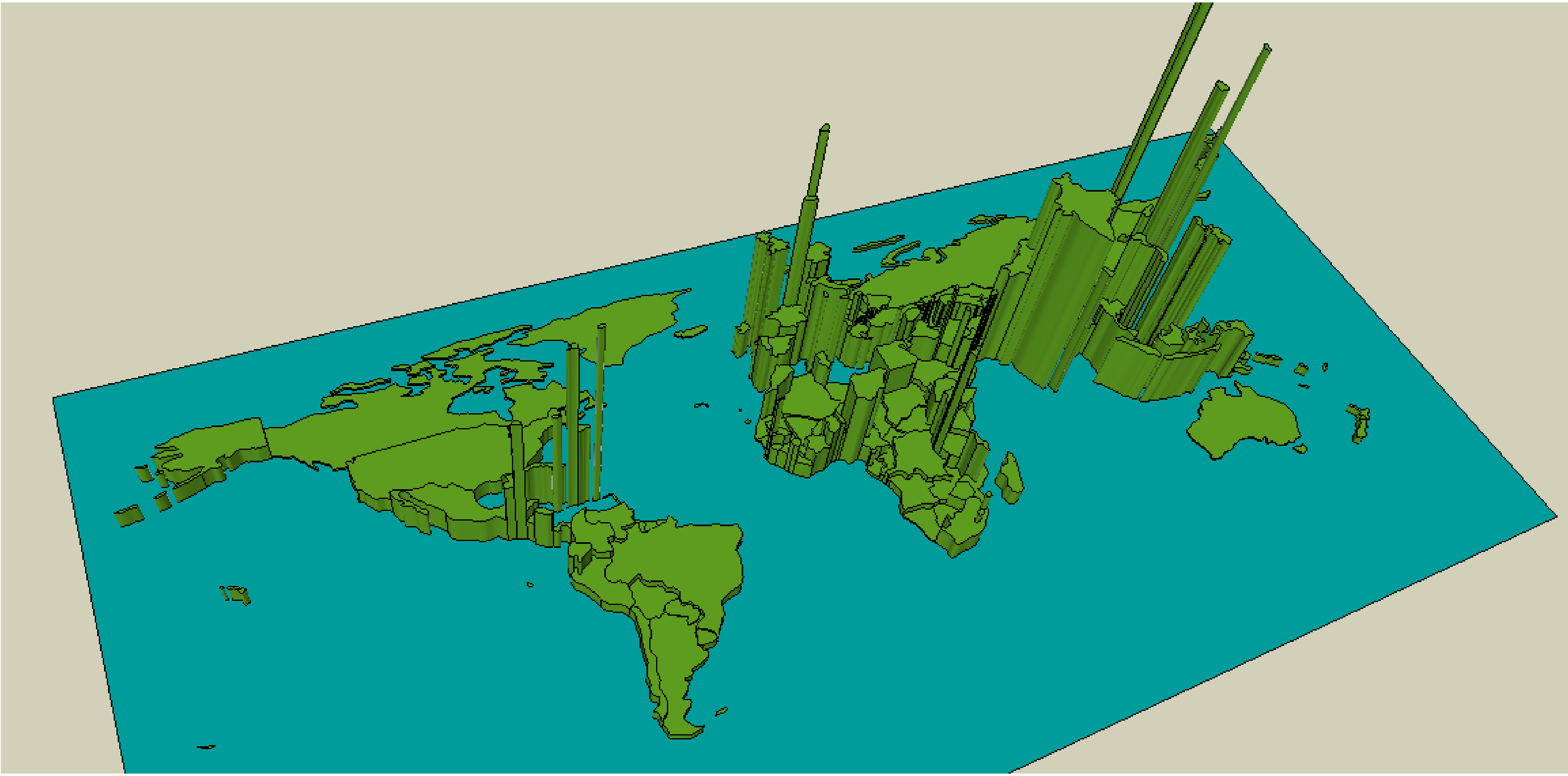
[http://i.imgur.com/gBYMfWO.jpg]

Thailand's Population Density

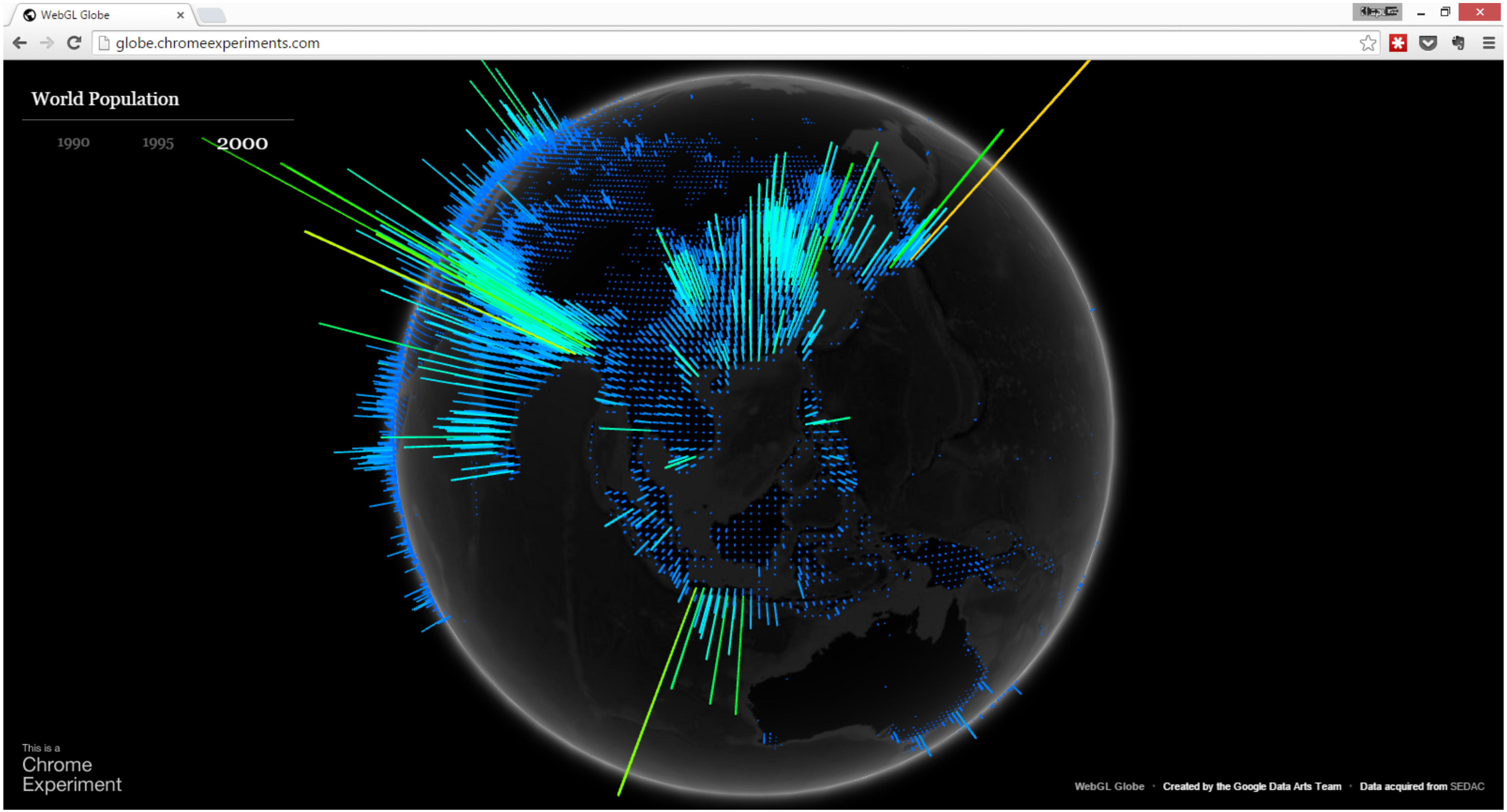


https://www.researchgate.net/publication/260378246_Climate-Related_Hazards_A_Method_for_Global_Assessment_of_Urban_and_Rural_Population_Exposure_to_Cyclones_Droughts_and_Floods/figures?lo=1

World Map of Population Density



World Map of Population Density



http://globe.chromeexperiments.com/

“Density”

- Density = quantity per unit of measure.
- Population Density = number of people per unit area
  - Location with high density value means there are a lot of people around that location.
  - Given a region, we integrate the density over that region to get the number of people residing in that region.
- Probability Density = probability per unit “length”.
  - Given an interval, we integrate the density over that interval to get the probability that the RV will be in that interval.

References

- From Discrete to Continuous Random Variables: [Y&G] Sections 3.0 to 3.1
- PDF and CDF: [Y&G] Sections 3.1 to 3.2
- Expectation and Variance: [Y&G] Section 3.3
- Families of Continuous Random Variables: [Y&G] Sections 3.4 to 3.5

Course Outline

The following is a tentative list of topics with their corresponding chapters from the textb
Yates and Goodman. Each topic spans approximately one week.

1.	Introduction, Set Theory, Classical Probability	[1]
2.	Combinatorics: Four Principles and Four Kinds of Counting Problems	[1]
3.	Probability Foundations	[1]
4.	Event-based Conditional Probability	[1]
5.	Event-based Independence	[1]
6.	Random variables, Support, Probability Distribution	[2]
7.	MIDTERM: 3 Oct 2019 TIME 15:00 - 17:00	
8.	Discrete Random Variables	[2]
9.	Families of Discrete Random Variables and Introduction to Poisson Processes	[2,10]
10.	Real-Valued Functions of a Random Variable	[2]
11.	Expectation, Moment, Variance, Standard Deviation	[2]
12.	Continuous Random Variables	[3]
13.	Families of Continuous Random Variables and Introduction to Poisson Processes	[3,10]

- Exercise 17 Solution [Posted @ 5PM on C
- References: [Y&G] Chapter 2
- Notes from the tutorial session [Posted @ 11:3
- Part IV: Continuous Random Variables
  - Chapter 10: Continuous Random Variables [Pos
    -
    - References
      - From Discrete to Continuous Rand
      - PDF and CDF: [Y&G] Sections 3.1 to
      - Expectation and Variance: [Y&G] Se
      - Families of Continuous Random Va

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10.2 Properties of PDF and CDF

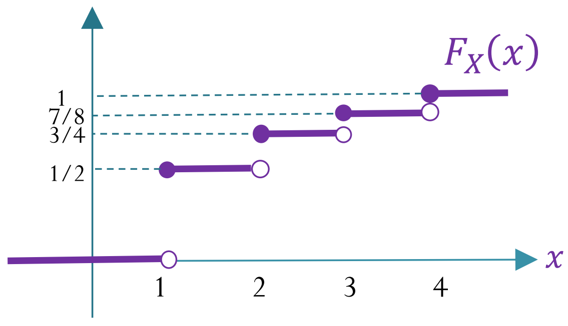
Sections 10.1-10.2

Discrete RV

- **pmf**: $p_X(x) \equiv P[X = x]$
  - Two characterizing properties:
    - $p_X(x) \geq 0$
    - $\sum_x p_X(x) = 1$
- $S_X = \{x: p_X(x) > 0\}$
- $P[\text{some condition(s) on } X]$

$$= \sum_{\substack{\{\text{all the } x \text{ values that} \\ \text{satisfy the condition(s)}\}}} p_X(x)$$

- **cdf** is a staircase function with jumps whose size at $x = c$ gives $P[X = c]$.

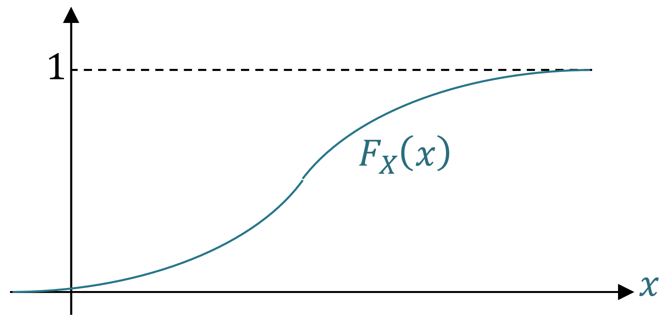


Continuous RV

- $P[X = x] = 0$
- **pdf** : $P[x_0 \leq x \leq x_0 + \Delta x] \approx \underbrace{f_X(x_0)}_{\text{probability per unit length}} \Delta x$
  - Two characterizing properties:
    - $f_X(x) \geq 0$
    - $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$
- $S_X = \{x: f_X(x) > 0\}$
- $P[\text{some condition(s) on } X] =$

$$\int_{\substack{\{\text{all the } x \text{ values that} \\ \text{satisfy the condition(s)}\}}} f_X(x) dx$$

- **cdf** is a continuous function.



pdf and cdf for continuous RV

$$F_X(b) - F_X(a)$$

$$P[a \leq X \leq b]$$

$$F_X(x) \equiv P[X \leq x]$$

$$P[a < X < b]$$

$$P[a \leq X < b]$$

$$P[a < X \leq b]$$

$$P[a \leq X \leq b]$$

$$\text{“}\frac{d}{dx}\text{”} F_X(x)$$

$$\int_{-\infty}^{x} f_X(t)\,dt$$

$$\int_{a}^{b} f_X(x)\,dx$$

$$f_X(x)$$

Finding Probabilities from CDF

Definition: $F_X(x) \equiv P[X \leq x]$

For **any RV**,	For **continuous RV**,
• $P[X \leq b] = F_X(b)$ $P[X < b] = F_X(b) - P[X = b]$	• $P[X \leq b] = F_X(b)$ $P[X < b] = F_X(b)$
• $P[X > a] = 1 - F_X(a)$ $P[X \geq a] = 1 - F_X(a) + P[X = a]$	• $P[X > a] = 1 - F_X(a)$ $P[X \geq a] = 1 - F_X(a)$
• $P[a < X \leq b] = F_X(b) - F_X(a)$	• $P[a < X \leq b] = F_X(b) - F_X(a)$ $P[a < X < b] = F_X(b) - F_X(a)$ $P[a \leq X < b] = F_X(b) - F_X(a)$ $P[a \leq X \leq b] = F_X(b) - F_X(a)$
• $P[X = a] = F_X(a) - F_X(a^-)$ (amount of jump in the CDF @ a)	• $P[X = a] = 0$

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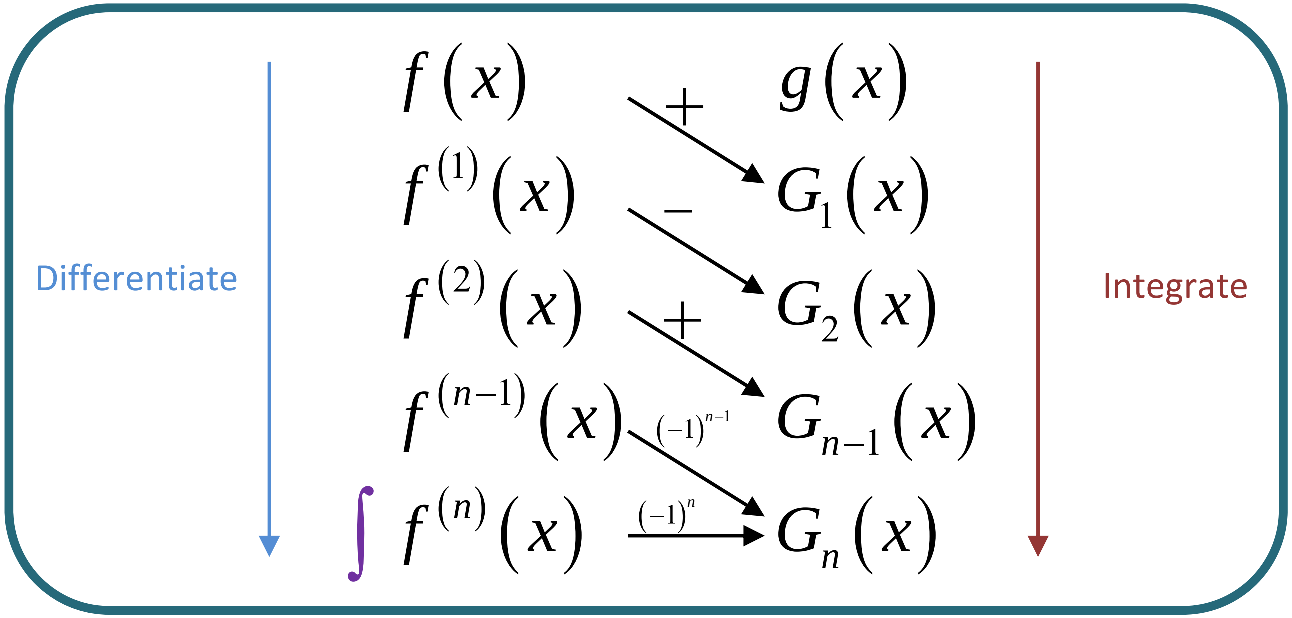
10.3 Expectation and Variance

Integration by Parts

- A technique for simplifying integrals of the form

$$\int f(x)g(x)\,dx$$

- **Tabular integration by parts**: A convenient method for organizing repeated application of integration by part:



$$\int f(x)g(x)\,dx = f(x)G_1(x) + \sum_{i=1}^{n-1}(-1)^i f^{(i)}(x)G_{i+1}(x) + (-1)^n \int f^{(n)}(x)G_n(x)dx + C$$

$$f^{(i)}(x) = \frac{d^i}{dx^i}f(x)$$

$$G_1(x) = \int g(x)\,dx$$

$$G_{i+1}(x) = \int G_i(x)\,dx$$

Integration by Parts

$$\int x^2 e^{3x}\,dx = \left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{27}\right)e^{3x} + C$$

$$= x^2\left(\frac{1}{3}e^{3x}\right) - \int 2x\,\frac{1}{3}e^{3x}\,dx$$

	Sign	
x^2	+	e^{3x}
$\int$ $2x$	−	$\frac{1}{3}e^{3x}$
2	+	$\frac{1}{9}e^{3x}$
$\int$ 0	−	$\frac{1}{27}e^{3x}$

$$\int (\sin x)e^x\,dx$$

$$= (\sin x - \cos x)e^x - \int (\sin x)e^x\,dx$$

$$= \frac{1}{2}(\sin x - \cos x)e^x + C$$

	Sign	
$\sin x$	+	e^x
$\cos x$	−	e^x
$\int$ $-\sin x$	+	e^x

Chapter 9 vs. Section 10.3

Discrete RV	Continuous RV
$\mathbb{E}X = \sum_x x p_X(x)$	$\mathbb{E}X = \int_{-\infty}^{\infty} x f_X(x) dx$
$\mathbb{E}[g(X)] = \sum_x g(x) p_X(x)$	$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$
$\mathbb{E}[X^2] = \sum_x x^2 p_X(x)$	$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x) dx$

$$\mathrm{Var}[X] = \mathbb{E}\left[(X - \mathbb{E}X)^2\right] = \mathbb{E}[X^2] - (\mathbb{E}X)^2$$

$$\sigma_X = \sqrt{\mathrm{Var}[X]}$$

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10.4 Families of Continuous Random Variables

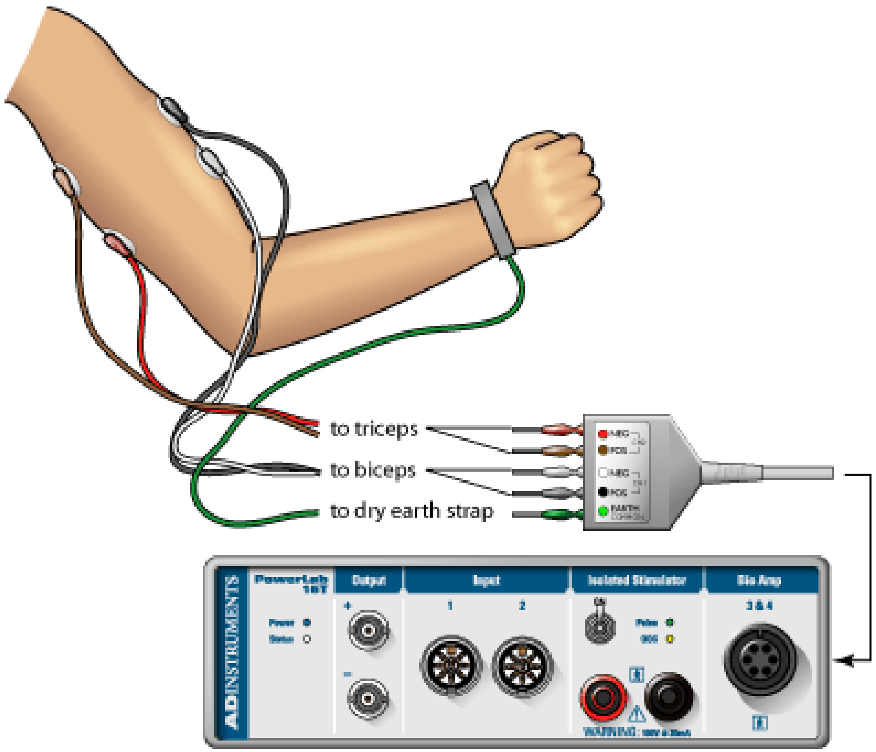
Johann Carl Friedrich Gauss

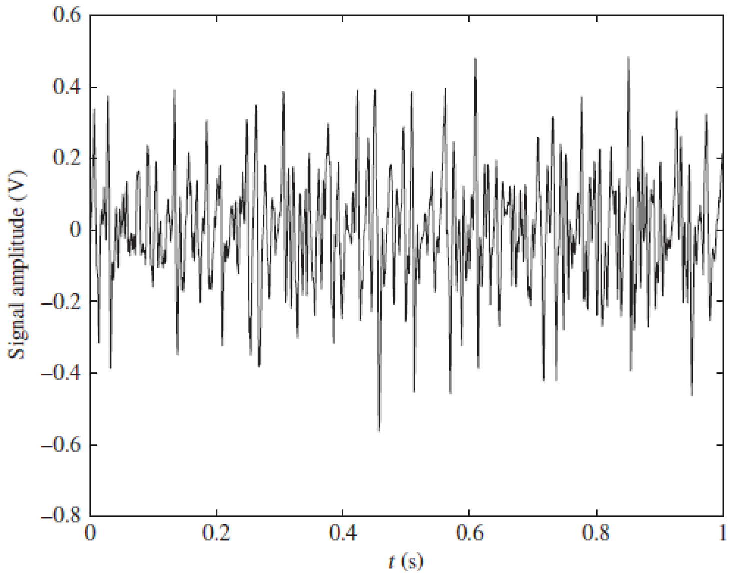
German 10-Deutsche Mark Banknote (1993; discontinued)

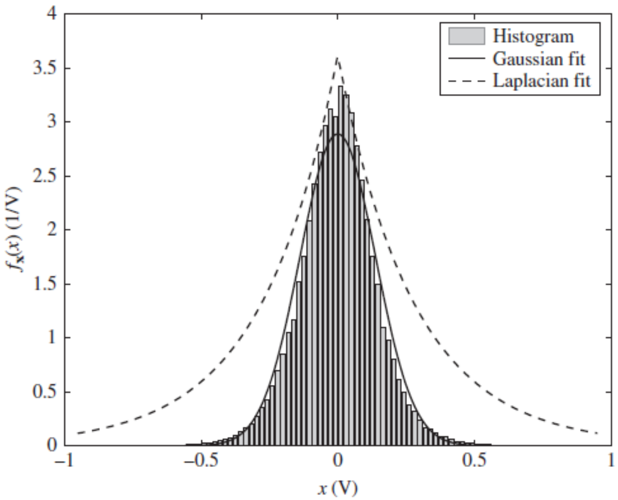
- 1777 –1855
- A German mathematician

Ex. Muscle Activity

- Look at electrical activity of skeletal muscle by recording a human electromyogram (EMG).



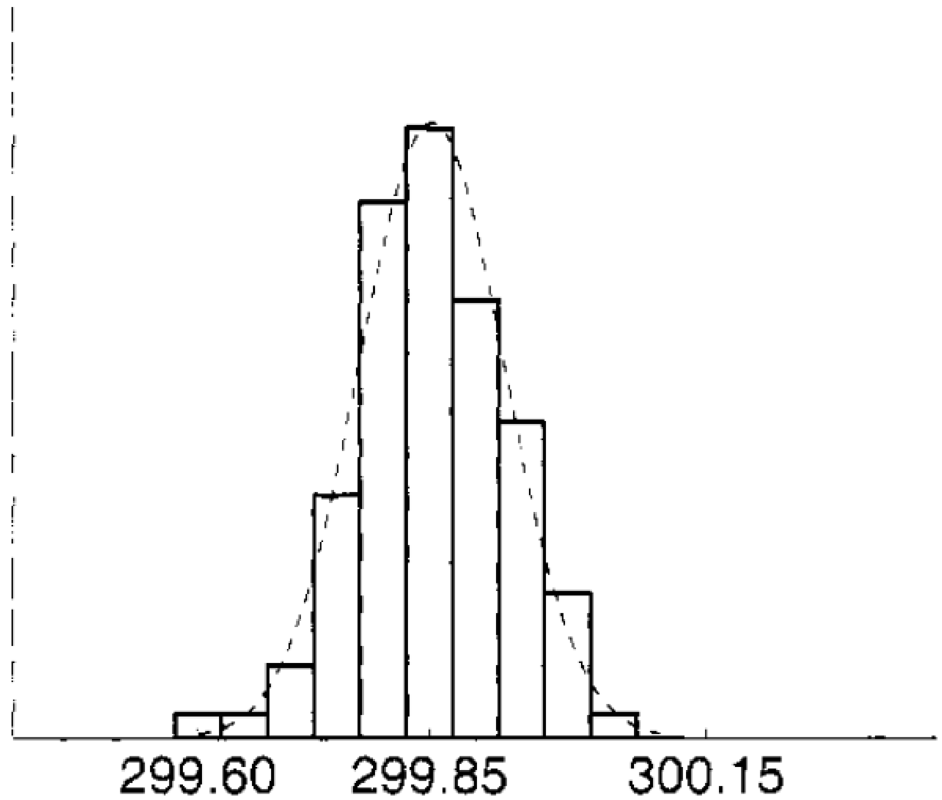




[http://www.adinstruments.com/solutions/education/ltexp/electromyography-emg-german]

Ex. Measuring the speed of light

- 100 measurements of the speed of light (×1,000 km/second), conducted by Albert Abraham Michelson in 1879.

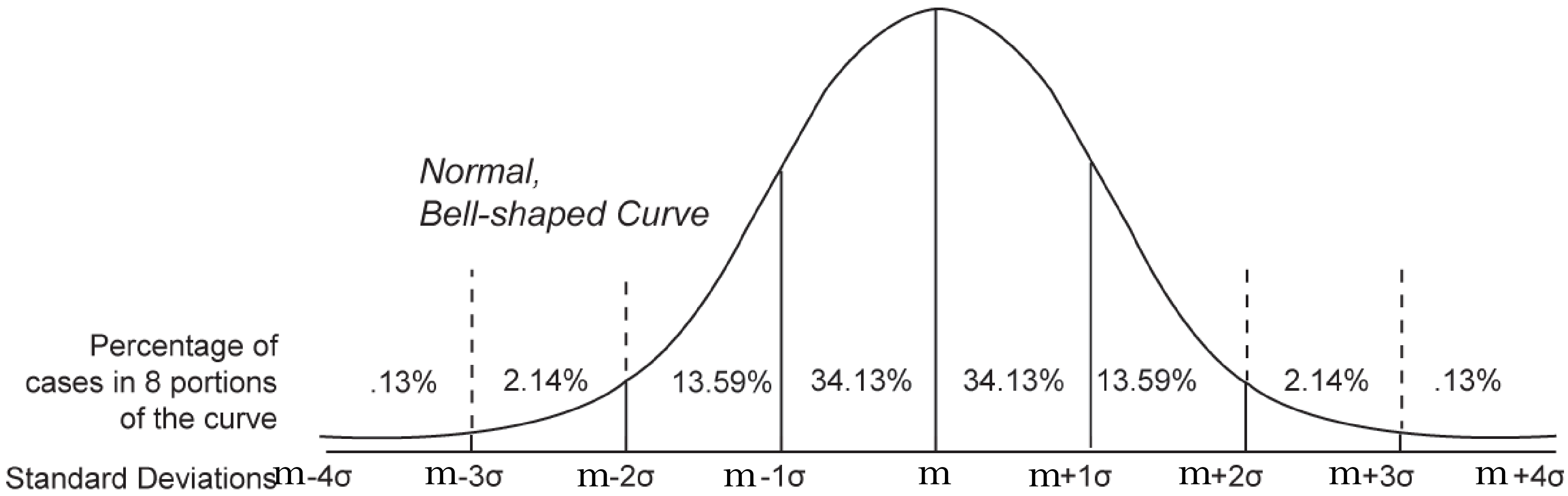


Expected Value and Variance

"Proof" by MATLAB's symbolic calculation

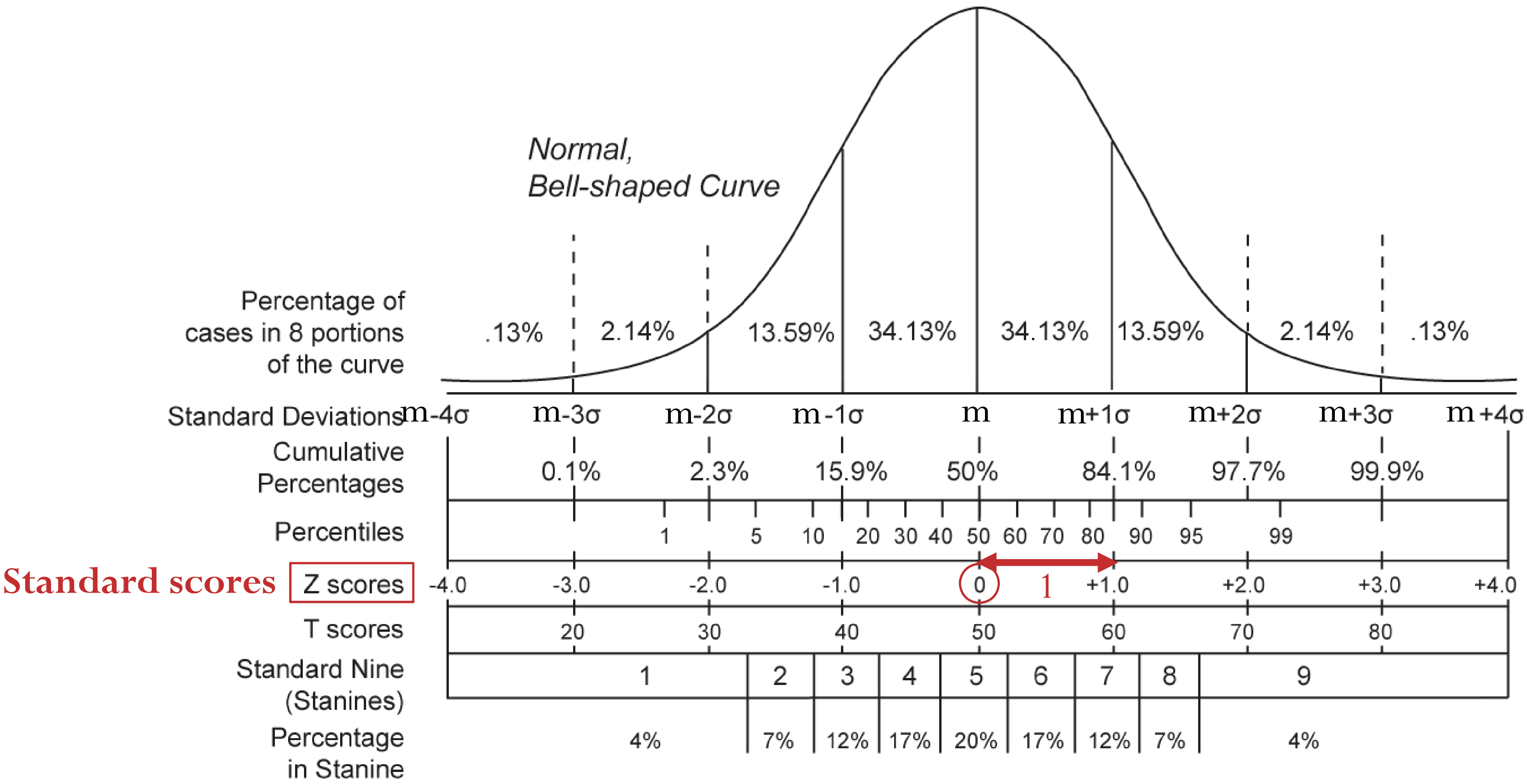
```
>> syms x
>> syms m real
>> syms sigma positive
>> int(1/(sqrt(sym(2)*pi)*sigma)*exp(-(x-m)^2/(2*sigma^2)),x,-inf,inf)
ans =
1
>> EX = int(x/(sqrt(sym(2)*pi)*sigma)*exp(-(x-m)^2/(2*sigma^2)),x,-inf,inf)
EX =
m
>> EX2 = int(x^2/(sqrt(sym(2)*pi)*sigma)*exp(-(x-m)^2/(2*sigma^2)),x,-inf,inf)
EX2 =
-(2^(1/2)*(limit(- x*sigma^2*exp((x*m)/sigma^2 - m^2/(2*sigma^2) - x^2/(2*sigma^2)) - m*sigma^2*exp((x*m)/sigma^2 - m^2/(2*sigma^2) - x^2/(2*sigma^2)) -
(2^(1/2)*pi^(1/2)*sigma*erfi((2^(1/2)*(x - m)*i)/(2*sigma))*(m^2 + sigma^2)*i)/2, x == -Inf) - limit(- x*sigma^2*exp((x*m)/sigma^2 - m^2/(2*sigma^2) -
x^2/(2*sigma^2)) - m*sigma^2*exp((x*m)/sigma^2 - m^2/(2*sigma^2) - x^2/(2*sigma^2)) - (2^(1/2)*pi^(1/2)*sigma*erfi((2^(1/2)*(x - m)*i)/(2*sigma))*(m^2 +
sigma^2)*i)/2, x == Inf)))/(2*pi^(1/2)*sigma)
>> EX2 = simplify(EX2)
EX2 =
m^2 + sigma^2
>> VarX = EX2 - (EX)^2
VarX =
sigma^2
```

Gaussian Random Variable



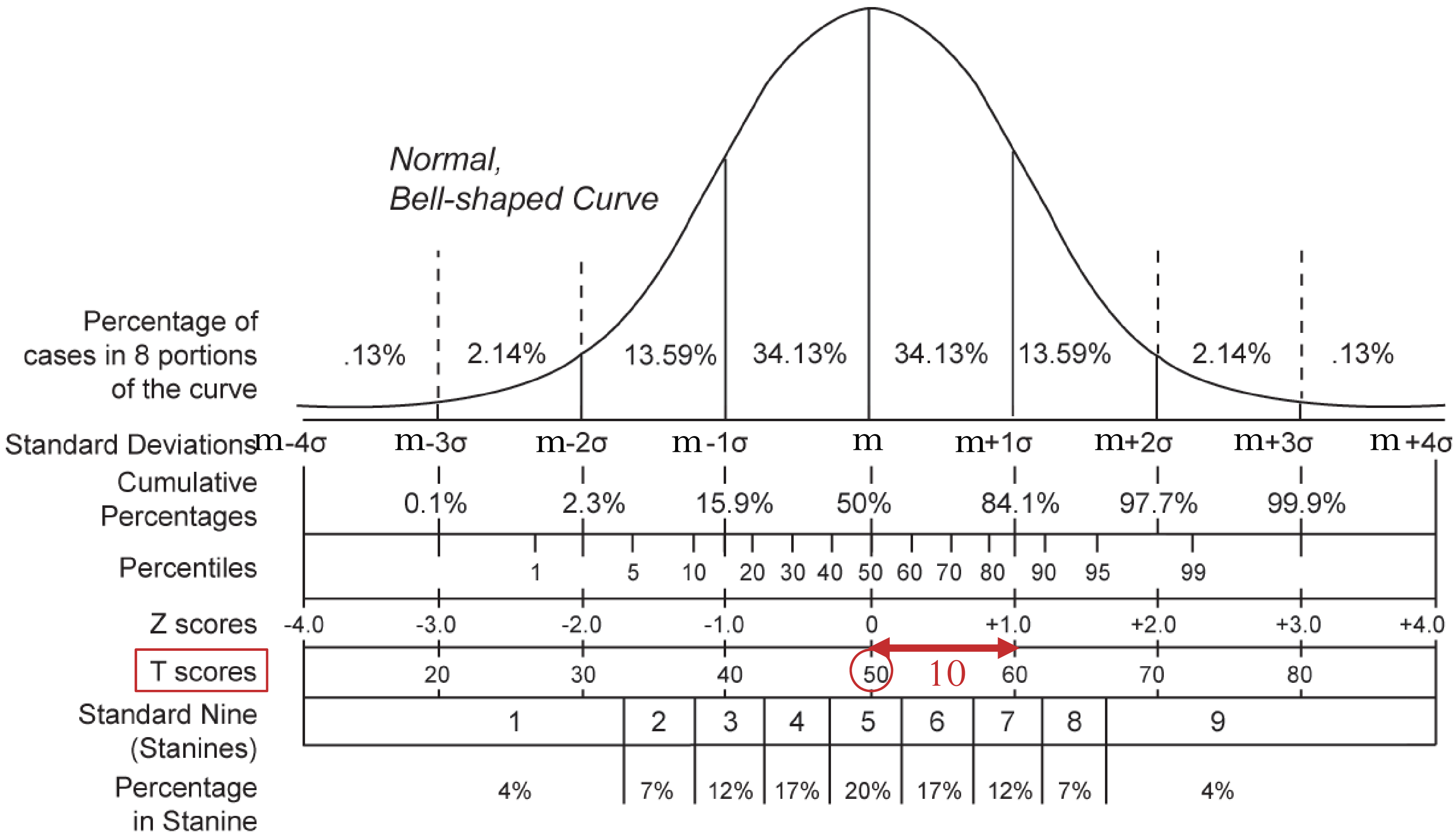
[Wikipedia.org]

Gaussian Random Variable



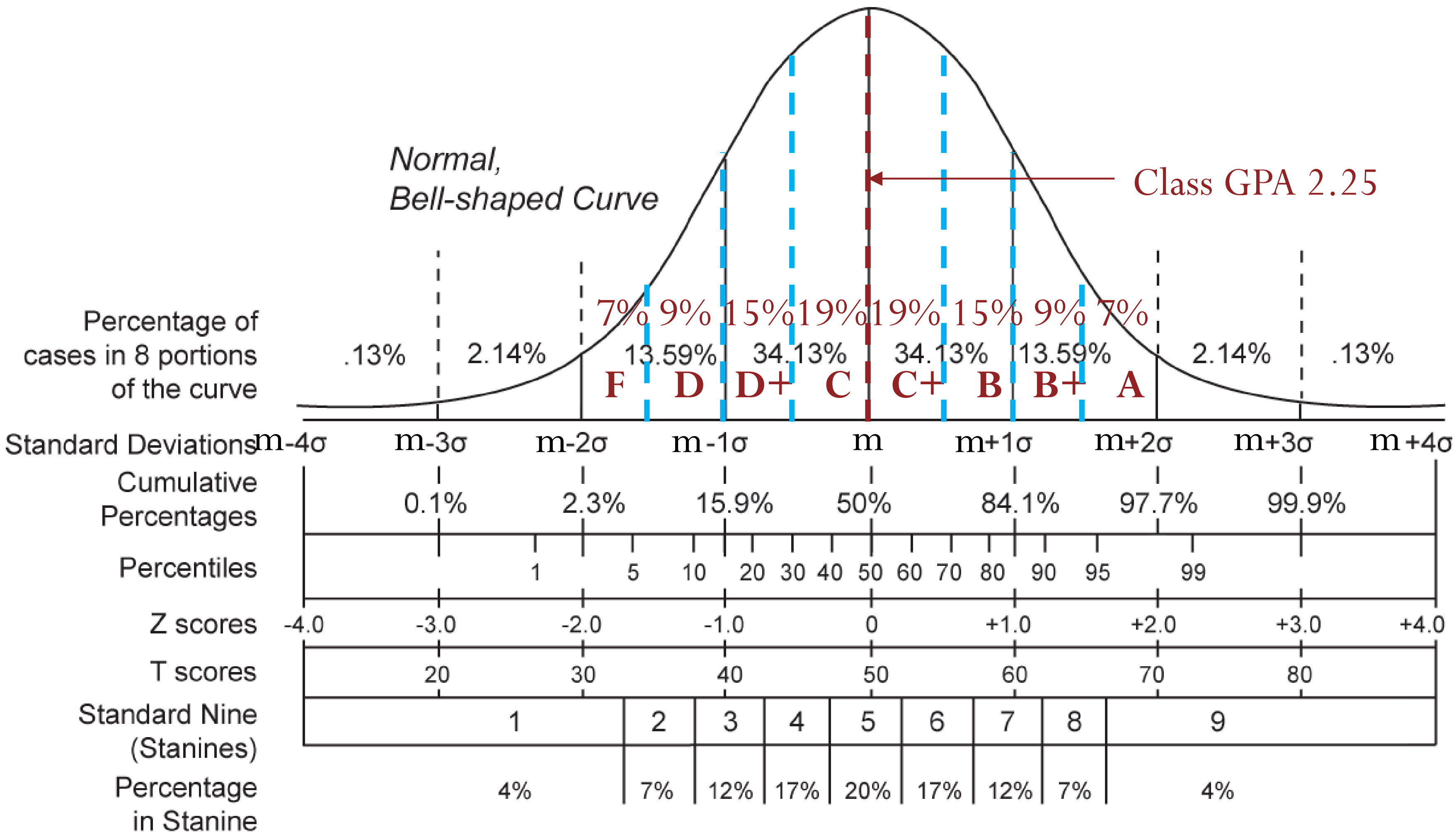
[Wikipedia.org]

Gaussian Random Variable



[Wikipedia.org]

SIIT Grading Scheme (Option 3)



[Wikipedia.org]

From the News

Higgs boson-like particle discovery claimed at LHC

COMMENTS (1665)

By Paul Rincon
Science editor, BBC News website, Geneva

4 July 2012

$$\frac{1}{1-\Phi(5)} \approx 3.5\times10^{6}$$

$$\frac{1}{1-\Phi(4.9)} \approx 2\times10^{6}$$

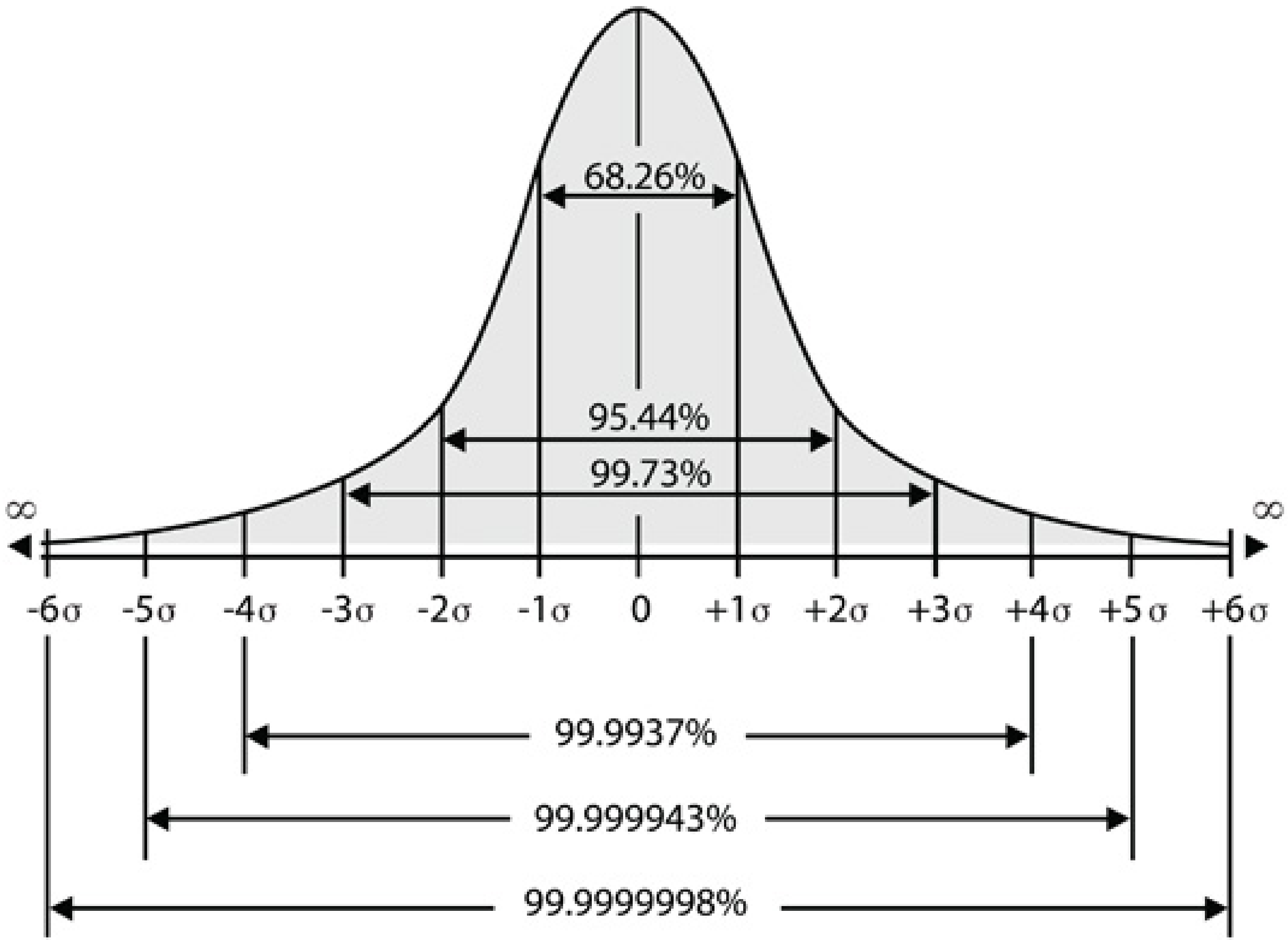
Particle physics has an accepted definition for a **discovery**: a "five-sigma" (or five standard-deviation) level of certainty

The number of sigmas measures how unlikely it is to get a certain experimental result as a matter of chance rather than due to a real effect

They claimed that by combining two data sets, they had attained a confidence level just at the "five-sigma" point - about a **one-in-3.5 million chance** that the signal they see would appear if there were no Higgs particle.

However, a full combination of the CMS data brings that number just back to **4.9 sigma** - a one-in-two million chance.

Six Sigma



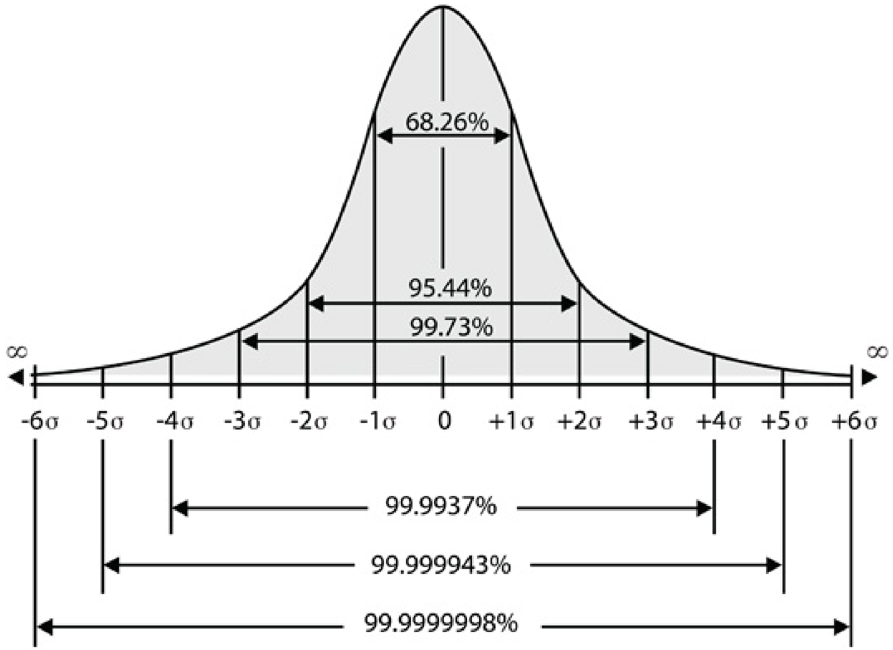
68.26%
95.44%
99.73%
∞
∞
-6σ
-5σ
-4σ
-3σ
-2σ
-1σ
0
+1σ
+2σ
+3σ
+4σ
+5σ
+6σ
99.9937%
99.999943%
99.9999998%

Six Sigma

- If you **manufacture** something that has a normal distribution and get an observation outside six σ of μ, you have either seen something extremely unlikely or there is something wrong with your manufacturing process. You'd better look it over.
- This approach is an example of **statistical quality control**, which has been used extensively and saved companies a lot of money in the last couple of decades.
- The term **Six Sigma**, a registered trademark of **Motorola**, has evolved to denote a methodology to monitor, control, and improve products and processes.
- There are Six Sigma societies, institutes, and conferences.
- Whatever Six Sigma has grown into, it all started with considerations regarding the normal distribution.

[Olofsson, 2006, p. 168]

Six Sigma



Range around μ	Percentage of products in conformance	Percentage of nonconforming products
-1σ to $+1\sigma$	68.26	31.74
-2σ to $+2\sigma$	95.46	4.54
-3σ to $+3\sigma$	99.73	0.27
-4σ to $+4\sigma$	99.9937	0.0063
-5σ to $+5\sigma$	99.999943	0.000057
-6σ to $+6\sigma$	99.9999998	0.00000002

[Bass, 2007, p. 20]

Probabilities involving Gaussian RV

- There is no closed-form simplification for

$$\int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx. \quad \text{(except for some special cases)}$$

- We have a table which gives the cdf of a standard Gaussian RV:

$$\Phi(z) \equiv F_Z(z) \text{ when } Z \sim \mathcal{N}(0,1).$$

  - The Φ table gives $\Phi(z)$ for $z \in [0,3)$.
  - Can use the property

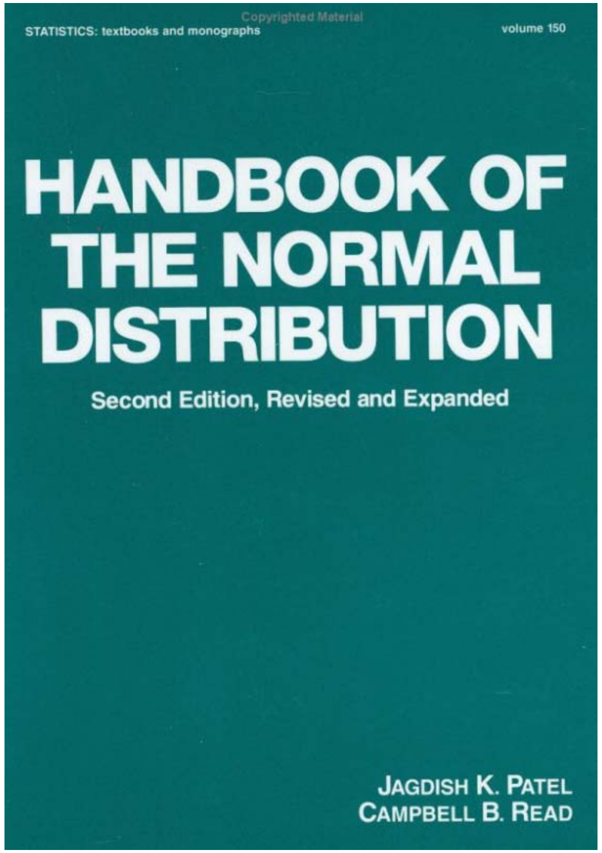
$$\Phi(-z) = 1 - \Phi(z)$$

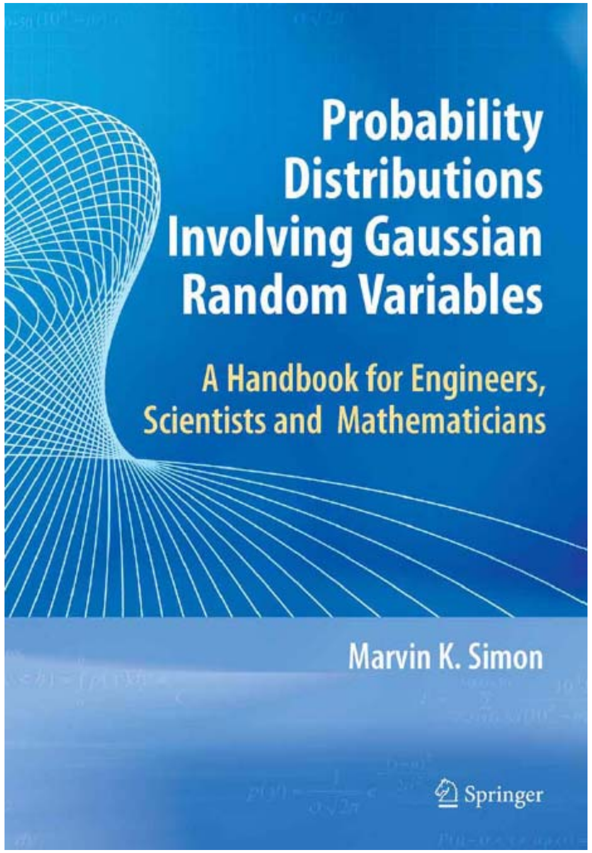
    to work with $z < 0$

Probabilities involving Gaussian RV

- There is no closed-form simplification for $\displaystyle\int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$.
  (except for some special cases)
- We have a table which gives the cdf of a standard Gaussian RV:
  $$\Phi(z) \equiv F_Z(z) \text{ when } Z \sim \mathcal{N}(0,1).$$
  - The Φ table gives $\Phi(z)$ for $z \in [0,3)$.
  - The Q table gives $Q(z) = 1 - \Phi(z)$ for $z \in [3,5)$.
  - Can use the property $\Phi(-z) = 1 - \Phi(z)$ to work with $z < 0$
- For $X \sim \mathcal{N}(m, \sigma^2)$,
  - $P[X \leq b] = P[X < b] = F_X(b) = \Phi\left(\frac{b-m}{\sigma}\right)$
  - $P[X > a] = P[X \geq a] = 1 - F_X(a) = 1 - \Phi\left(\frac{a-m}{\sigma}\right)$
  - $P[a < X \leq b] = P[a < X < b] = P[a \leq X < b] = P[a \leq X \leq b]$
    $= F_X(b) - F_X(a) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right)$

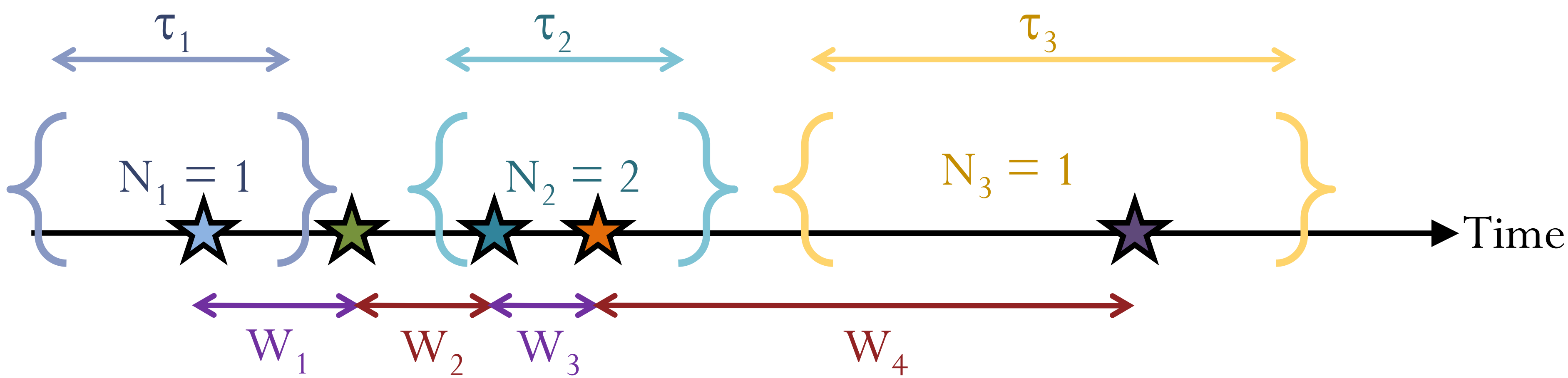
More on Gaussian RVs…





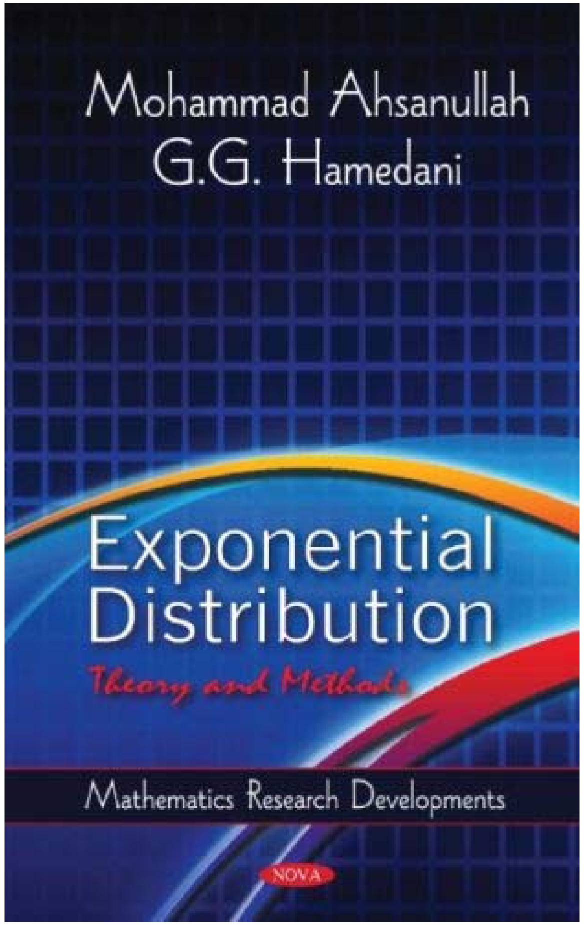
Poisson Process

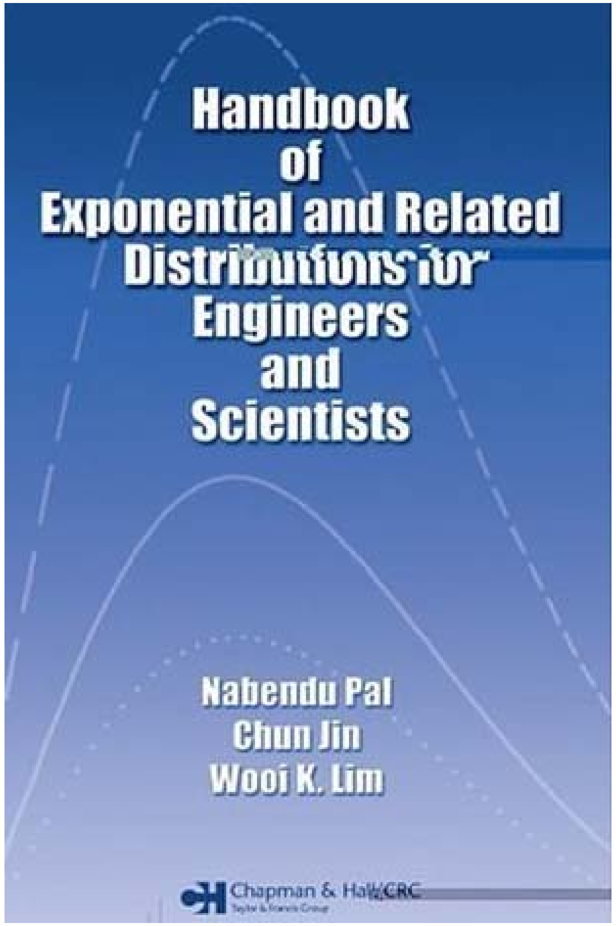
The number of arrivals $N_1, N_2, N_3, \ldots$ during non-overlapping time intervals are independent Poisson random variables with mean $= \lambda \times$ the length of the corresponding interval.



The lengths of time between adjacent arrivals $W_1, W_2, W_3, \ldots$ are i.i.d. exponential random variables with mean $1/\lambda$.

More on Exponential RV …





References

- From Discrete to Continuous Random Variables: [Y&G] Sections 3.0 to 3.1
- PDF and CDF: [Y&G] Sections 3.1 to 3.2
- Expectation and Variance: [Y&G] Section 3.3
- Families of Continuous Random Variables: [Y&G] Sections 3.4 to 3.5

Course Outline

The following is a tentative list of topics with their corresponding chapters from the textb
Yates and Goodman. Each topic spans approximately one week.

1. Introduction, Set Theory, Classical Probability [1]
2. Combinatorics: Four Principles and Four Kinds of Counting Problems [1]
3. Probability Foundations [1]
4. Event-based Conditional Probability [1]
5. Event-based Independence [1]
6. Random variables, Support, Probability Distribution [2]
7. **MIDTERM: 4 Oct 2018 TIME 09:00 - 11:00**
8. Discrete Random Variables [2]
9. Families of Discrete Random Variables and Introduction to Poisson Processes [2,10]
10. Real-Valued Functions of a Random Variable [2]
11. Expectation, Moment, Variance, Standard Deviation [2]
12. Continuous Random Variables [3]
13. Families of Continuous Random Variables and Introduction to Poisson Processes [3,10]

- Excercise 15 Solution [Posted @ 4:30PM on N
- Excercise 16 Solution [Posted @ 3PM on Nov
- Slides [Posted @ 4:30PM on Nov 6]
- Part IV: Continuous Random Variables
  - Chapter 10 [Posted @ 10AM on Nov 5]
    - Annotated notes for Sections 10.1-10.3 [Pos
    - References
      - From Discrete to Continuous Random
      - PDF and CDF: [Y&G] Sections 3.1 to 3.2
      - Expectation and Variance: [Y&G] Sectio
      - Families of Continuous Random Variab
- Part V: Multiple Random Variables

Probability and Random Processes

ECS 315

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10.5

Review: Function of discrete RV

Example 9.16. Let

$$p_X(x) = \begin{cases} \frac{1}{10} x^2, & x = \pm 1, \pm 2 \\ 0, & \text{otherwise} \end{cases}$$

$\mathbb{E}X = 0$

and

$$Y = X^4.$$

Find $p_Y(y)$ and then calculate $\mathbb{E}Y$. $= \sum_y y\, p_Y(y)$

Step 1: Find c

$$\sum_x p_X(x) = 1$$

$$\frac{1}{c}\left(1^2 + 2^2 + (-1)^2 + (-2)^2\right) = 1$$

$c = 10$.

Step 2: Find $p_Y(y)$

Note that $Y = X^4$

$p_X(x)$	x	y
1/10	1	$1^4 = 1$
1/10	−1	$(-1)^4 = 1$
4/10	2	$2^4 = 16$
4/10	−2	$(-2)^4 = 16$

$P[Y=1] = P[X=1] + P[X=-1] = 2/10 = 1/5$

$P[Y=16] = P[X=2] + P[X=-2] = 8/10 = 4/5$

$$p_Y(y) = \begin{cases} 1/5, & y = 1, \\ 4/5, & y = 16, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbb{E}Y = \sum_y y\, p_Y(y) = 1 \times \frac{1}{5} + 16 \times \frac{4}{5} = \frac{65}{5} = 13$$

References

- From Discrete to Continuous Random Variables: [Y&G] Sections 3.0 to 3.1
- PDF and CDF: [Y&G] Sections 3.1 to 3.2
- Expectation and Variance: [Y&G] Section 3.3
- Families of Continuous Random Variables: [Y&G] Sections 3.4 to 3.5
- SISO: [Y&G] Section 3.7; [Z&T] Section 5.2.5